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Emergence of Constraint in Self-organizing Systems

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Abstract: *Practitioners of agent-based modeling are often tasked to model or design self-organizing systems while the theoretical foundation of self-organization in science remains to be set. This paper explores self-organization in the context of an agent-based model of ant colony food foraging. We gather specific measures of order-creation and constraint construction particular to leading theories of nonequilibrium thermodynamics that purport to govern self-organizing dynamics. These measures are used to explore three claims: (a) Constraints are constructed from entropy-producing processes in the bootstrapping phase of self-organizing systems; (b) positive feedback loops are critical in the structure formation phase; and (c) constraints tend to decay. The continued presence of far-from-equilibrium boundary conditions are required to reinforce constraints in the maintenance phase.*

Key Words: constraint, self-organization, entropy, ant simulation, agent-based modeling.

INTRODUCTION

Most agent researchers study organization of one type or another. Example organizations of scientific interest include Rayleigh-Bénard convection, lasers, cellular slime molds, immune systems, genetic regulatory networks, neural systems and insect colonies. Organizations of more commercial interest include firms, supply chains,

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financial markets, transportation systems, manufacturing production lines and IT systems. Despite the difference of detail in modeling domains, agent researchers share a common goal of understanding how the coordination of semi-autonomous components of a nonlinear dynamical system may spontaneously emerge as a system moves toward global attractor states. (Ashby, 1957; Haken, 2000; Kauffman, 1993; Kelso, 1995; Kugler & Turvey, 1987; Prigogine, 1984).

The increasing variety of agent-based modeling (ABM) software toolsets (Gilbert & Bankes, 2002) and the accelerating availability of serious desktop computing power make modeling accessible to researchers with minimal investments in specialized programming skills and hardware expense. Nonetheless, Eric Bonabeau (2002) observes, "although ABM is technically simple, it is also conceptually deep. This unusual combination often leads to improper use of ABM" (p. 7280).

Improper use is not entirely the fault of an uninformed practitioner. Core concepts in the field of ABM remain ill defined. For example, Physics, Biology, Chemistry, Cognitive Science, Economics, Sociology and Ecology all have substantial literature related to self-organizing phenomena in their respective fields. However, despite their efforts as a collective enterprise, Science continues to lack a rigorous and generally accepted definition of "organization" (Kauffman, 2000, 2003; Rashevsky, 1960; Rosen, 1991).

Promisingly, new (and not so new) theories claim to resolve fundamental issues of organization (Atkins, 1984; Brooks & Wiley, 1986; Kauffman, 2000; Kugler & Turvey, 1987; Prigogine, 1962, 1984; Schneider and Kay, 1994; Swenson, 1989; Tsallis, 1998; Ulanowicz, 1986). These somewhat overlapping theories seek to explain the emergence of organization as an expected consequence of driving constraints forcing systems far from thermodynamic equilibrium. Bill McKelvey (in press) offers a review of some of these theories and focuses on their strengths and weaknesses with respect to order-creation. Additionally, Francis Heylighen (2002) provides a good introduction to related concepts central to self-organization.

In the past, the ABM community has borrowed liberally from Thermodynamics and Statistical Mechanics, two research methodologies in physics used to understand ensembles of interacting components. For example, it is common for researchers to talk in terms of the *microscopic* agent-level and the *macroscopic* system behavior (Bonabeau, 2002; Epstein & Axtell, 1996). As systems self-organize, changes in macroscopic states are expressed in terms of *phase transitions, correlation lengths, mean free paths* and *mean relaxation times* (Kauffman, 1993, 1995, Kugler & Turvey, 1987; Swenson & Turvey, 1991, Theraulaz et al, 2002).

Mapping nonequilibrium thermodynamic concepts onto ABM may prove more tractable if we start with *thin agent* models. As a rough distinction, agent research can be separated into modeling that uses *fat agents* and modeling that uses thin agents. Fat agents typically have sophisticated internal reasoning, large memory and complex learning algorithms. Fat agent systems often have relatively few agents with some agents having access to global information aggregated from many agents or from a large portion of the interaction space. These agent systems can be said to characterize much of the work in Distributed Artificial Intelligence (DAI) and Multi-Agent Systems (MAS). Most of the complexity of the model is internal to individual agents making thermodynamic accounting difficult.

Thin agents, by contrast, make decisions using simple rule bases and have limited to no memory. They perceive mostly local information in their spatial neighborhood and communicate with a few agents out of the possible agent population at any one time. The environment is often modeled explicitly as an active process and plays a significant role in model evolution (Parunak, 1997). Thin agents are typically associated with the modeling of complex adaptive systems and self-organizing systems. This approach can be said, albeit with some municipal hubris, to characterize the "Santa Fe Approach". A common theme is that the emergent collective intelligence of complex adaptive systems resides not in complex individual abilities but rather in networks of agent-agent and agent-environment interactions. Our model of an ant colony constructing shortest paths to food sources is an example case. The ants as agents have extremely simple rules of gradient-following and pheromone dropping. No individual ant is capable of learning the location of the food source. Learning and intelligence takes place in the colony – environment system taken as a whole.

An idea put forth in this paper is that agent-agent and agent-environment interactions along with internal rules of agents can be considered forms of constraint. From our perspective, organization can be measured, in part, as a bundle of self-reinforcing and evolving constraints.

ORIGIN AND MAINTENANCE OF CONSTRAINT

The intent of this paper is to take a few small steps in exploring some ideas common to the nonequilibrium thermodynamics models. Three claims of interest are:

1. Constraints can be constructed from entropy-producing processes in the *bootstrapping phase* of self-organizing systems.
2. Positive feedback loops are critical in the *structure formation phase*.
3. Constraints tend to decay. The continued presence of far-from-equilibrium boundary conditions are required to reinforce constraints in the *maintenance phase*.

These three points relate to how systems may learn the structure of their environment. As will be demonstrated, learning can be measured as an increase of constraints that limit degrees of freedom of agents (Kugler & Turvey, 1987). Specifically, learning in ABM can occur through (a) changes in agent interaction patterns e.g. edge weighting in neural networks (Rumelhart & McClelland, 1986), (b) changes in agents' internal rules, e.g. distributed classifier systems (Holland, 1995) or (c) changes to potential information stored in the environment, e.g. pheromone trail following in ant foraging systems (Bonabeau, Dorigo & Theraulaz, 1999). Our model uses this last, *stigmergic* form of learning. However, these principles should equally apply to the first two forms of learning.

EXPERIMENTAL SETUP

The following simulation of food gathering ants is presented for the purpose of calculating statistical and thermodynamic measures that help characterize phases of self-organization. The spirit of this model is an extension of the work of Parunak and Brueckner (2001) and Gutowitz (1993).

The ant system described here is discrete; the positions of all objects in the system are specified by a 2-tuple of integers (x, y). The space of positions is a square grid. The three types of objects are nests, food, and ants. Ants and the environment are modeled as active agents in the simulation. Additionally, each position in the space can contain some amount of nest pheromone and food pheromone, which are deposited by the ants as they move.

A basic simulation is set up as follows: A nest and some amount of food are placed in the space. A fixed number of ants is initially placed at the nest. All positions have zero levels of both food and nest pheromones. The system evolves as the ants move, drop pheromone, and transport food. The model is flexible to later allow experimentation of initially placing ants at any location in the space and to allow any type of object (nests, food, ants) to be introduced at any time or position.

An ant can hold one unit of food at a time and can take one of three actions: (a) move to one of eight adjacent locations (includes diagonal moves), (b) pick up a unit of food, and (c) drop a unit of food at a nest. The following pseudo-code describes what actions an ant will take on each time step:

```
if ant has food then
    drop one unit of food pheromone
    if at nest then
        drop food
    else
        follow nest pheromones
    end if
else
    drop one unit of nest pheromone
    if at food then
        pick up food
    else
        follow food pheromones
    end if
end if
```

Also, each time step some percentage of the pheromone present at each position “evaporates”, or is removed. Pheromone evaporation allows adaptation to changes in food location. For example, if there were two food sources present, *A* and *B*, and the ants were exploiting *A* for a period of time a strong trail of pheromones would be laid between the nest and *A*. Once the food at *A* is gone the ants should no longer follow that trail, but should rather explore again to find *B*. If the pheromones leading to *A* do not evaporate this cannot occur. The decay or forgetting of constructed constraints (pheromone trails) allows the system to be adaptive.

The ants have directionality. They can only travel to their forward five positions instead of choosing from all eight adjacent positions. This local directionality is present regardless of the state of the system and is an example of a *context-free constraint* (Juarrero, 1999). The direction of an ant is calculated after each step based on the previous and current positions. At time zero each ant chooses a random direction.

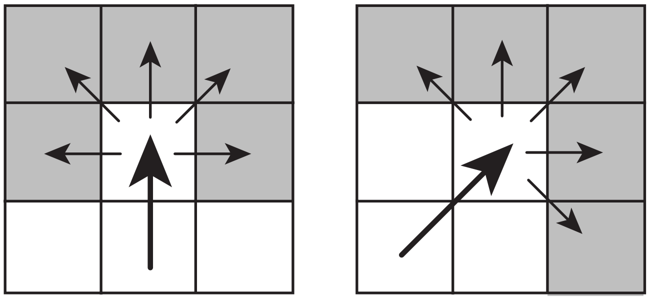

Fig. 1. Examples of possible next steps (in gray) for an ant according to the last step taken. Ants have hard context-free constraints preventing backwards movement.

Each time step ants measure a local gradient then choose a direction to step. Pheromone levels are read from the forward five positions. The probability of moving to position *j* is given by:

$$p_j = \frac{\mu_j^{\alpha} + \beta}{\sum_{n=1}^{N} \mu_n^{\alpha} + \beta} \qquad (1)$$

where μ_j is the pheromone level at position *j*; α is a scaling exponent; β is a random base; and the denominator represents the total pheromone level in all possible next positions. In this case *N* is five. The scaling exponent α increases the probability that the next position will be the one with the greatest pheromone level, whereas the random base β has the opposite effect. Typical values used in our experiments were $\alpha = 3$, $\beta = 1$, and a maximum pheromone level of 511 at any one position. These parameters can be adjusted to tune the likelihood that an ant will explore for new food versus exploiting a found food source.

So, the movement of an ant is constrained by a measure of change not an absolute strength. One can think of the gradient as the spatial first derivative of the pheromone field.

Figure 2 graphically depicts four phases of the typical evolution of this ant system. In the next section, measures of constraint and spatial entropy are defined as tools for examining the construction and destruction of constraints in this self-organizing system.

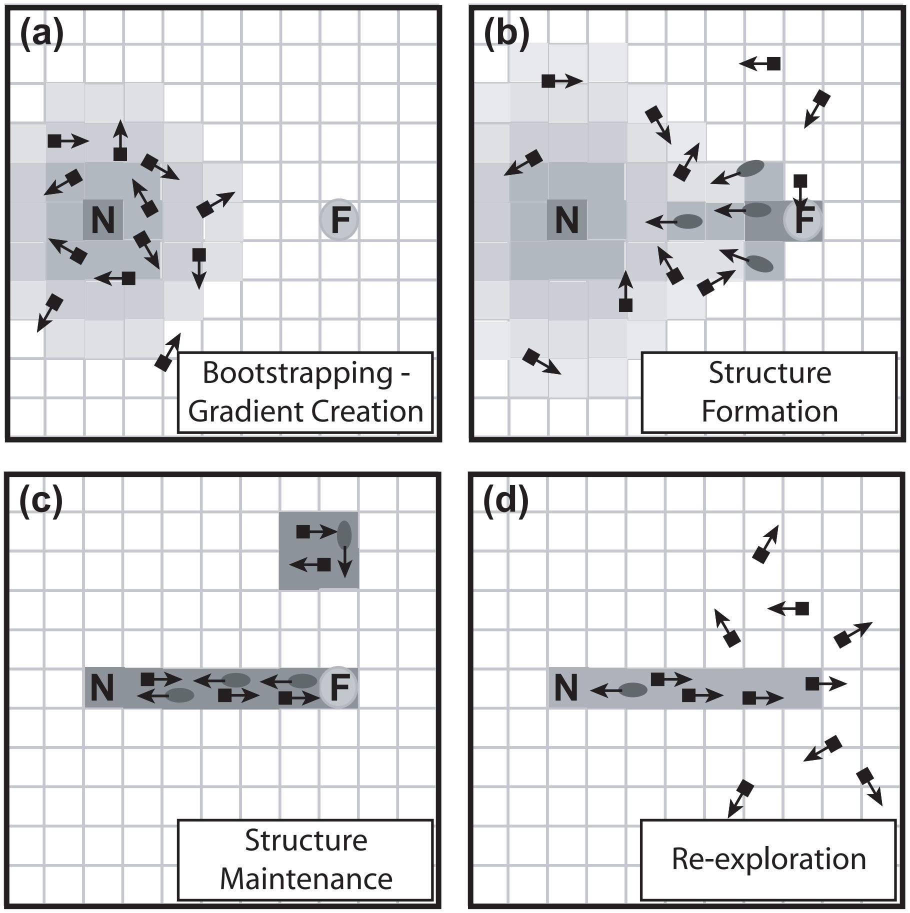

Fig. 2. Typical evolution of the ant system. (a) *Bootstrapping – Gradient Creation:* Ants move randomly out from the nest, creating a gradient of nest pheromones. (b) *Structure Formation:* Some ants find the food and begin following the nest pheromones while dropping food pheromones that food-seeking ants begin to follow. (c) *Structure Maintenance:* A stable path of both food and nest pheromones is established. As shown in the upper-right corner, cycles that do not transport food can also form. (d) *Re-exploration:* Once all of the food has been transported to the nest the pheromones begin to evaporate and the ants disperse.

CONSTRAINT AND SPATIAL ENTROPY MEASURES

As a system self-organizes, components of the system are expected to lose degrees of freedom through the emergence of context-sensitive constraints (Juarrero, 1999). In this system, ants lose directional degrees of freedom as they are informed by a gradient. We measure this constraint in our model with a directional entropy. An ant that sees no pheromone gradient, which is an equal level of pheromone in all possible next positions, is said to be maximally ignorant and has an ignorance level of 1. An ant that has no choice but to move to one specific position on the next step would have an ignorance of 0, though this never occurs here because of the random base added to each pheromone level as described in Eq. 1. The Shannon entropy (Shannon, 1948a, 1948b) of the probabilities of moving to each of the possible positions on the next step defines the ignorance for each ant

$$S = \frac{-\sum_{n=1}^{N} p_n \log p_n}{\log N} \tag{2}$$

where p_n is the probability of moving to position n, and N is the number of possible next positions, in this case five. The denominator is used to normalize the value to the range [0,1]. The ignorance of a group of ants is defined as the average ignorance of all ants.

As systems self-organize, statistical measures of order increase. In this model, we capture increased order with a spatial entropy measure applied to the positions of ants. The spatial entropy measure is also a Shannon entropy of the form shown in Eq. 2 where p_n is the proportion of all ants at position n, and N is the total number of positions in the space. Note that in the case of zero ants being at a location 0 log 0 = 0. The maximum spatial entropy is achieved with an equal number of ants at each position and the minimum with all ants at a single location.

EXPERIMENTAL RESULTS

The results presented here will be used to support the following three points. These points are model-specific restatements of the three ideas from nonequilibrium thermodynamics put forth earlier.

1. In the bootstrapping phase of the ant model, an increase in spatial entropy leads to a decrease in the ignorance of the ants returning

to the nest. This increase in entropy enables later phases of structure formation and maintenance.

2. Structure is created by a positive feedback loop of decreasing ant spatial entropy and increasing constraint (decreasing ignorance) on ant movement. A decrease in the former causes, and is caused by, an increase in the latter.

3. The structure is dependent on the presence of sufficient amounts of food. The presence of a separated nest and concentrated food source can be seen as a far-from-equilibrium external constraint necessary for the maintenance of structure.

All results given here are based on an average over 20 runs with the same initial conditions. The space was 21 positions square, with a single nest at position *(7,7)*, a single food source at position *(15,15)*, and all ants starting at the nest.

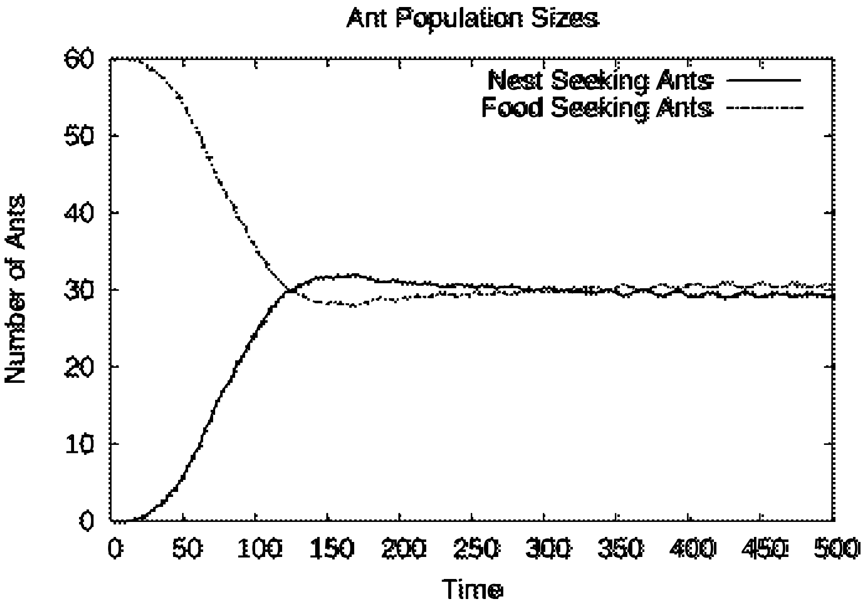

Fig. 3. Relative population sizes of nest seeking and food seeking ants. As the system matures the population sizes reach equilibrium.

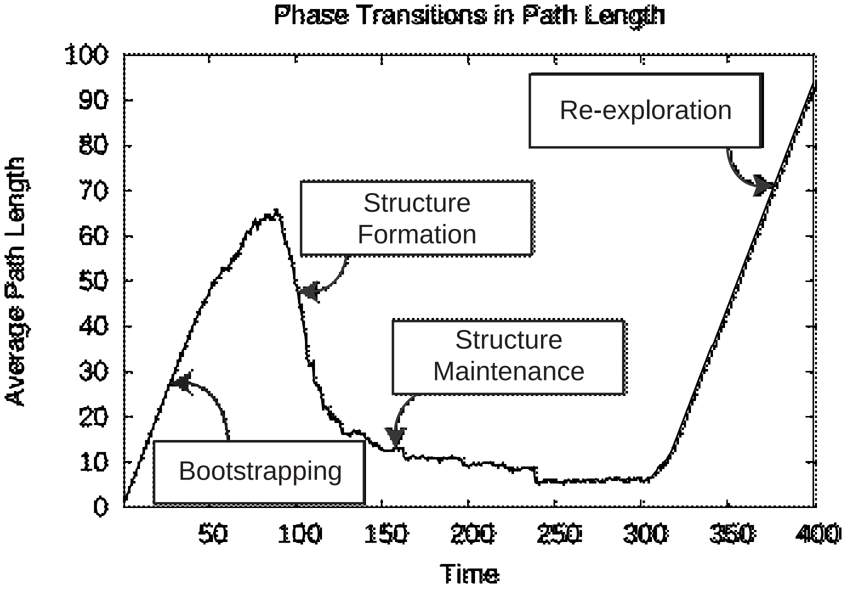

Fig. 4. The four phases of development are visible in this plot of the ants' mean path length at each time step. A path length is calculated as the number of steps an ant has taken since it last picked up or dropped a unit of food.

As an example, Fig. 3 shows the number of food seeking and nest seeking ants averaged over 20 runs of a system with 60 ants, all starting at the nest, over 500 time steps. As would be expected, the number of each type of ant reaches equilibrium once a path between the nest and the food is established.

An indicator of which phase the system is in (i.e. order parameter) could be the mean path length of the ants. The path length of an ant at a given time is defined as the number of steps it has taken since it last picked up or dropped a unit of food. Figure 4 shows the four phases of development with a plot of mean path length vs. time. In the bootstrap phase the mean path length increases uniformly. The line in this case has a slope of one because each ant takes one step per unit time and no ants have yet found food. The structure formation phase begins when food is found which leads to a rapid decrease in the mean path length. During the structure maintenance phase, mean path length remains stable at a near minimum value (the shortest distance between the nest and food). When the food source is depleted, the mean path length again increases as the ants re-explore the space for alternative food sources.

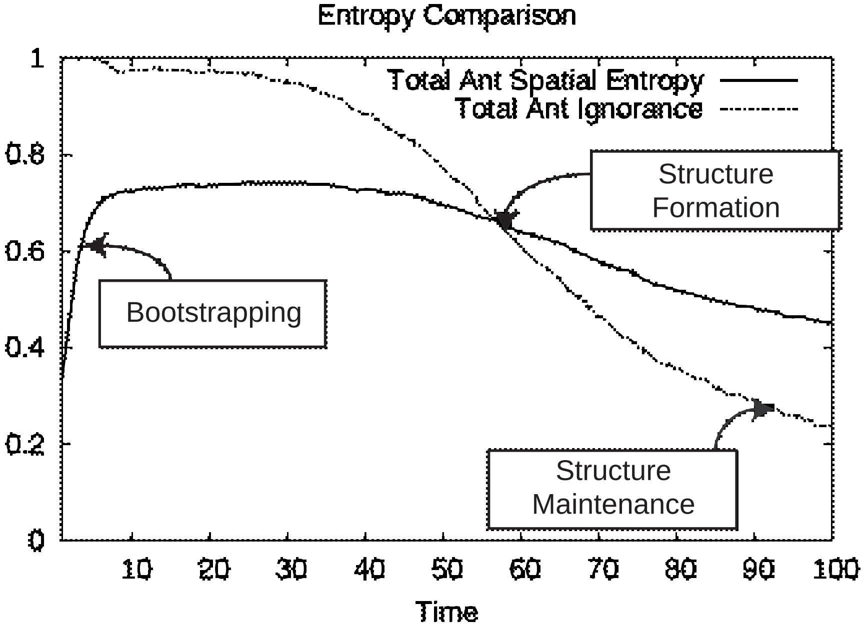

Fig. 5. A comparison of the spatial entropy and ignorance of ants over time. In the *bootstrapping – gradient creation* phase the spatial entropy of the ants increases, which establishes a gradient of pheromones around the nest. In the *structure formation* phase the ignorance of the ants decreases as they find food and begin to follow the nest pheromones back. The spatial entropy of the ants also begins to decrease when a path is formed between the nest and food. The *structure maintenance* phase begins when the spatial entropy and ignorance of the ants becomes relatively constant.

Figure 5 displays the spatial entropy and ignorance for all ants over the first 100 time steps of an experiment with an inexhaustible food source. The bootstrapping phase occurs over approximately the first 25 time steps. In this phase, the ants' random walk from the nest causes a rapid increase in spatial entropy and the establishment of a nest pheromone gradient around the nest. In the structure formation phase ants that find food use this gradient to direct their return to the nest. The food-carrying ants' constrained movement is reflected in a reduction of overall ant ignorance as the gradient informs them to the nest location.

Increasing spatial entropy causally constraining ant movement is offered as an illustration of Point (1): Constraints can be constructed from entropy-producing processes in the *bootstrapping phase* of self-organizing systems. This is also in agreement with Parunak and Brueckner's (2001) findings. They describe this effect as "coordination can arise through coupling the macro level (in which we desire agent self-organization with a concomitant decrease in entropy) to an entropy-increasing process at a micro level" (p. 130).

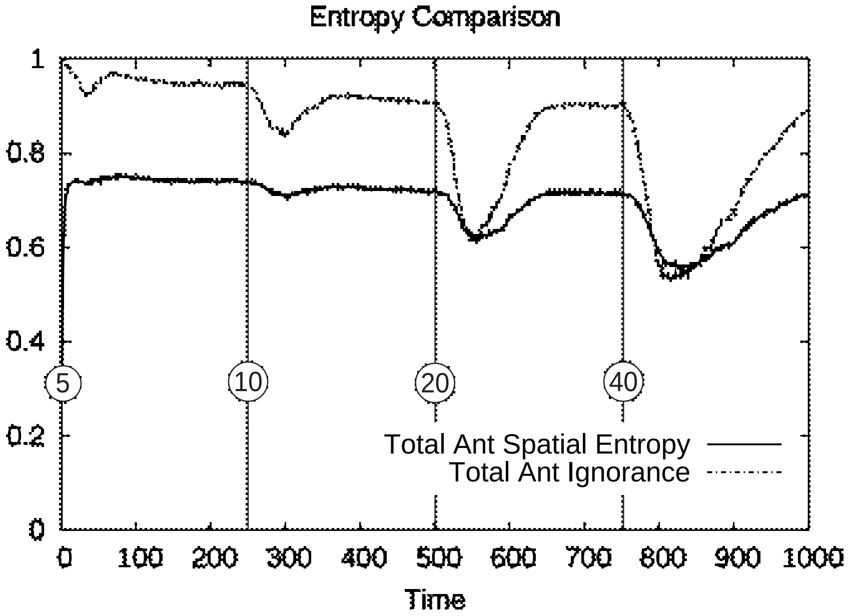

Fig. 6. Ant spatial entropy and ignorance over time as greater amounts of food are injected into the system. The vertical lines represent points at which food is injected (always at the same location). Food is injected every 250 time steps, starting with 5 food units and doubling each time. Increased order arises with larger injections of food, as shown by the more pronounced decreases in entropy.

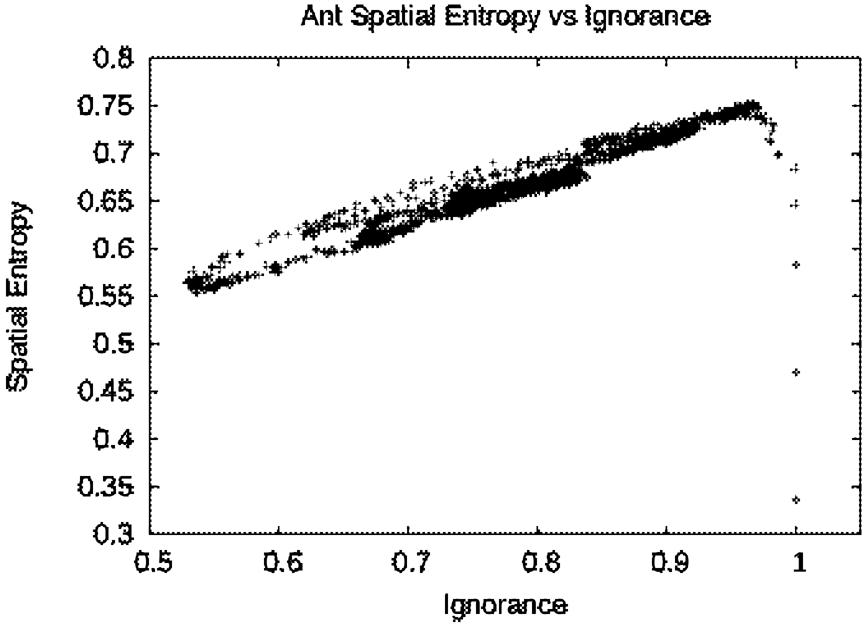

Fig. 7. Plot of ant spatial entropy vs. ignorance for the experiment shown in Fig. 6. A correlation coefficient of ρ=0.94 implies a link between the two. The rightmost outliers of high ignorance and low spatial entropy occur during the *bootstrapping* phase when ants leave the nest. The correlated points occur during the *structure formation*, *structure maintenance*, and *re-exploration* phases.

Figure 6 demonstrates the dependence of the structure formation and structure maintenance phases on a food source. In this case, limited amounts of food are injected every 250 time steps, starting with 5 food units at time zero and doubling the amount of food added each interval. In the presence of a concentrated food source, structure is created as shown by the decreasing spatial entropy and decreasing ignorance of the ants. Once that food source is depleted the structure breaks up as the constraints on the ants' movements (the pheromone field) gradually decay. This effect is offered as an illustration of Point (3): Constraints tend to decay. The continued presence of far-from-equilibrium boundary conditions are required to reinforce constraints in the *maintenance phase*.

There is a correlation between spatial entropy and ignorance in Fig 6. This correlation is due to the positive feedback loop between decreasing ant spatial entropy and increasing constraint on ant movement (decrease in ignorance). Figure 7 makes this correlation (ρ=0.94) more clear with a scatter plot of ant spatial entropy vs. ignorance. This relationship is put forth to illustrate Point (2): Positive feedback loops are critical to structure formation.

The few uncorrelated points occur during the bootstrapping phase as spatial entropy is increasing while ignorance remains at its maximum value.

SUMMARY

Three claims from nonequilibrium thermodynamics were explored in the context of an ant foraging agent-based model: (1) Constraints can be constructed from entropy-producing processes, (2) Positive feedback loops are critical to structure formation, and (3) The continued presence of far-from-equilibrium boundary conditions are required to reinforce internal constraints. We developed measures of constraint and order that illustrate these claims.

These initial findings can be considered first steps in establishing mappings from nonequilibrium thermodynamics to ABM. Next steps could explore various cycles present in this model as they may relate to thermodynamic work cycles which are considered necessary for Kauffman’s Autonomous Agents (2000, 2003). Candidate cycles in this model include (1) ant movement cycles between nest and food sources, and (2) system cycling through the phases of bootstrapping, structure formation, maintenance and re-exploration.

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