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A network flow model for lane-based evacuation routing

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Abstract

Most traffic delays in regional evacuations occur at intersections. Lane-based routing is one strategy for reducing these delays. This paper presents a network flow model for identifying optimal lane-based evacuation routing plans in a complex road network. The model is an integer extension of the minimum-cost flow problem. It can be used to generate routing plans that trade total vehicle travel-distance against merging, while preventing traffic crossing-conflicts at intersections. A mixed-integer programming solver is used to derive optimal routing plans for a sample network. Manual capacity analysis and microscopic traffic simulation are used to compare the relative efficiency of the plans. An application is presented for Salt Lake City, Utah.

Keywords: Emergency management; Evacuation routing; Network flow modeling; Optimization modeling

1. Introduction

Evacuation is a common strategy in emergency management. In many hazardous events, the best option is to relocate threatened populations to safer areas. This is a complex problem with many behavioral and management facets (Perry, 1985; Vogt and Sorensen, 1992; Dow and Cutter, 1998; Drabek, 1999). A zone to evacuate must be agreed upon (Sorensen et al., 1992), shelters and exits must be designated (Sherali et al., 1991), and evacuees must be routed to safety under dynamic hazard and traffic conditions (MacGregor-Smith, 1991; Southworth, 1991). Any number of transportation problems can arise during an evacuation. For example, notifying evacuees may be difficult, traffic delays are common, and transportation lifelines are often compromised by the hazard.

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A central challenge in managing a regional evacuation is routing people to safety. An efficient routing plan is valuable because evacuations routinely result in travel demand that exceeds the available network capacity. This can occur at scales from an individual neighborhood to an entire urban region, as in the Oakland–Berkeley Fire evacuation in 1991 and the Hurricane Floyd evacuation in 1999. Traffic delays may range from inconvenient to catastrophic. Important factors that affect the outcome of an evacuation include the time available before (or after) the hazard's impact, the expected travel demand, and the consequences of not clearing the area in a timely manner. As transportation infrastructure is a limited resource both in terms of directional accessibility and capacity, it is worth pursuing a routing plan that makes optimal use of this infrastructure.

Most traffic delays during an evacuation occur at intersections (Southworth, 1991). Lane-based routing is a valuable strategy for reducing these delays. In a lane-based routing plan, select turning options at intersections are restricted to improve traffic flow away from a hazardous area. A plan might require vehicles in the right lane of an intersection approach to turn right while requiring vehicles in the left lane to continue straight. One benefit of this type of routing is that intersections with potentially significant delays can be temporarily transformed into an uninterrupted flow facility. This increases intersection approach capacities in directions favorable for evacuating a defined area. Lane-based routing can also be used to reduce (or eliminate) intersection crossing and merging conflicts. In general, restricting intersection turning-latitude inconveniences some drivers in the name of increasing system-wide transportation network throughput.

The 2000 Cerro Grande Fire evacuation in Los Alamos provided a valuable example of lane-based routing (LAC, 2001). Evacuees north of the main transportation artery in Los Alamos, Diamond Boulevard, were instructed to enter the westbound *right lane* of this Boulevard and remain in that lane. Evacuees originating south of Diamond Boulevard were instructed to enter the westbound *left lane* and remain in that lane. For this reason, vehicles north of Diamond did not merge with vehicles from the south. Furthermore, major intersections were placed in a state of uninterrupted flow. This allowed emergency managers to evacuate more than 11,000 residents in a few hours with only one major exit, a feat that would have taken many more hours under normal traffic control.

This paper presents a network flow model for identifying optimal lane-based evacuation routing plans in a complex road network. We review prior work in prescriptive evacuation routing and outline criteria for identifying an optimal lane-based routing plan. A subsequent section presents the model formulation. A mixed-integer programming solver is used to derive routing plans for sample networks. Manual capacity analysis and microscopic traffic simulation are used to evaluate the efficiency of the plans. Lane-based routing plans are generated for an area in downtown Salt Lake City, Utah. The paper concludes with a discussion of the results and areas for further research.

2. Prescriptive evacuation routing

2.1. Prior work

There are few examples of prescriptive routing in regional evacuation modeling. Evacuation routing research has emphasized the dynamic flow of evacuees and route-choice modeling (Sheffi et al., 1982; Southworth, 1991; Hobeika et al., 1994; Pidd et al., 1996; Urbanik, 2000). Dunn and Newton (1992) proposed a maximum-flow approach to evacuation routing with the objective of moving the most flow (evacuees) from a source to a sink through a capacitated network (Ford and Fulkerson, 1962). An estimate of the population at source nodes is required along with the available links and their capacities. This approach is very valuable in assessing network capacity for long-term evacuation planning, but the results can be difficult to implement in practice. Traffic managers are limited in their ability to accurately estimate origin flow volumes during an emergency, although methods to address this problem have been proposed (Glickman, 1986). It is also difficult for emergency managers to precisely allocate flow volumes to links when drivers are generally free to select a route to a designated destination.

Yamada (1996) used the minimum-cost flow problem to assign pedestrian evacuees to shelters at the city scale. Yamada defined the shortest evacuation plan (SEP) as one where the total distance from all evacuees to all shelters is minimized. This approach can also be used in a road-network context, where each vehicle is routed to its nearest evacuation zone exit under a shortest network-distance assumption. This can be very valuable in assessing metrics like total travel distance for a given scenario. However, in a road-network context, assigning vehicles to their closest exit can result in significant merging and congestion. Also, a vehicular SEP can be difficult to implement because emergency managers must communicate *closest exits* and *shortest paths* in a complex road network to all drivers. In cases where it is possible to communicate this information using an intelligent transportation system (ITS), successful execution generally depends on a high level of voluntary participation in the plan.

Recently, Campos et al. (2000) presented a k-shortest path method for identifying optimal independent evacuation routes. In their method, an individual path is better if it has a greater capacity and lower travel time. The optimal set of disjoint routes between an origin and destination node is defined as the one with the greatest sum of the capacity/time ratios for each route. The application that Campos et al. refer to is identifying routes to evacuate the area surrounding a nuclear power plant. The scale of the analysis is the same as that of Dunn and Newton (1992) in that a node represents a relatively large traffic-generating region surrounding a nuclear power plant.

2.2. Lane-based routing

Although it is difficult to accurately estimate source flows into a network, allocate flows to links, and communicate best routes to evacuees in an emergency, it is relatively straightforward to control turning latitude at intersections. Traffic managers routinely improve vehicle flow away from a hazardous area by blocking entry into an evacuation zone and placing barriers to restrict lanes at intersections to turning left, right, or continuing straight. This is a complex network-based decision space. A first step toward representing it in a network optimization model is defining a set of criteria for rating lane-based evacuation routing plans.

A central goal in evacuation routing is to transform critical intersections into uninterrupted flow facilities. An uninterrupted flow facility does not require vehicles to come to a full stop. Fig. 1a depicts the conflict points in a standard four-leg single-lane approach intersection. Removing conflict points can reduce intersection delays and total evacuation time. The same action also reduces potential intersection accident points (Poch and Mannering, 1996; Rao and Rengaraju,

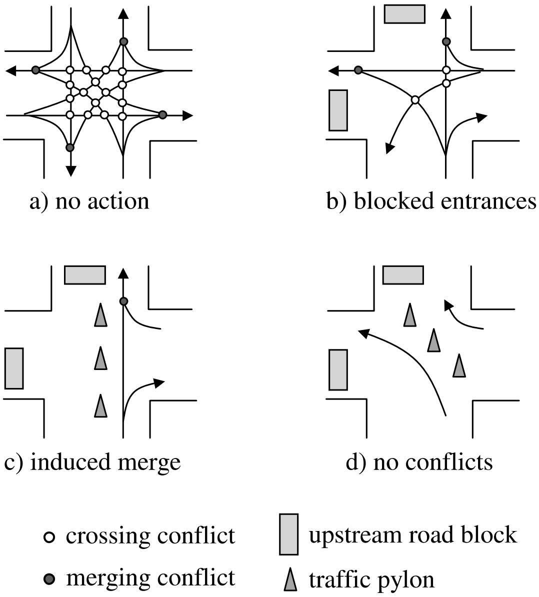


Fig. 1. Evacuation routing plans for a four-leg, single-lane appoach intersection with various crossing and merging conflict levels.

1997). Fig. 1b shows one example where 13 of the 16 crossing conflicts (81%) and 2 of the 4 merging conflicts (50%) in an intersection are eliminated by blocking traffic at two upstream intersections. This is analogous to permanently converting the two intersecting streets to one-way streets (ITE, 1993). Fig. 1c shows how traffic pylons (or barriers) can be set up to eliminate all crossing conflicts and further reduce merging conflicts to 1. Fig. 1d represents the ideal case where the pylons are configured to remove all intersection conflict points.

The prior example highlights two objectives for lane-based evacuation routing plans: minimizing intersection crossing conflicts and merging conflicts. A third objective in evacuation routing is to minimize lane changing along multi-lane arterials. If a significant percentage of vehicles changes lanes along a link, traffic delays and evacuation time may increase. It might not be possible to physically restrict lane changing during an evacuation due to time constraints in setting up temporary barriers, but instructions can be included in an evacuation order to achieve this end. An order might stipulate that, "Vehicles from the north should enter the right lane of Boulevard *a* traveling west and remain in that lane." Complete driver participation is not necessary, as any reduction in lane changing can reduce traffic delays.

Reducing the number of crossing and merging conflicts in an evacuation routing plan can increase the distance that vehicles must travel to reach an evacuation zone exit. Another objective is to minimize the total evacuee travel distance (i.e. shortest network distance). This is the shortest evacuation plan (SEP) criterion (Yamada, 1996). The SEP criterion establishes a fundamental trade-off between total travel-distance and merging. At one extreme, evacuees will be routed to their closest exit under a shortest network-distance assignment, but merging will be at its greatest. At the other extreme, merging is minimized, but evacuees may be routed away from their closest exit to achieve this end.

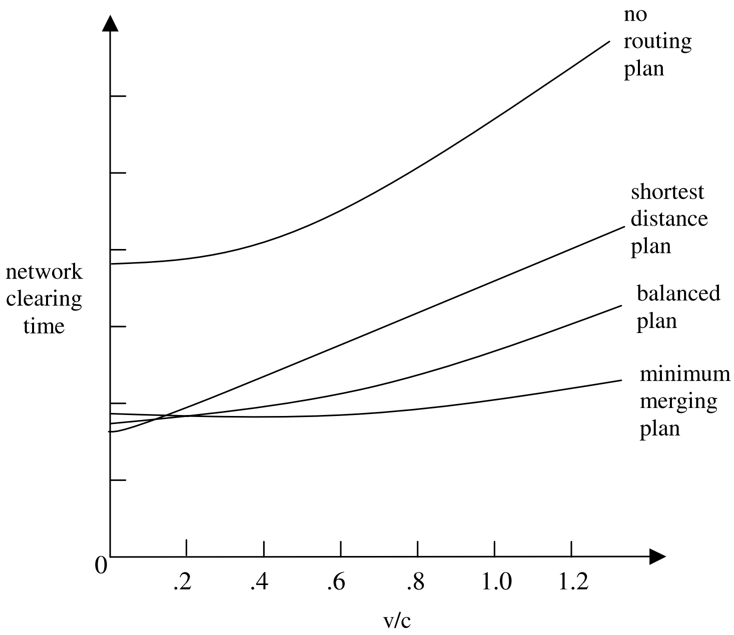


Fig. 2. The theoretical effect of removing intersection conflict points (crossing and merging) from an evacuation routing plan on network clearing time.

Fig. 2 depicts the expected effect of this trade-off on evacuation times under various traffic volume scenarios. Under heavy volumes, the decrease in network clearing time afforded by reducing the number of intersection crossing and merging conflicts should be greatest. Under moderate traffic volumes, the decrease would be less because the inherent delays in having traffic cross and merge is less. In scenarios with light traffic volumes, the trend in evacuation time reverses, and it becomes quicker to send vehicles to their closest exit (shortest network distance) because there is little to no delay associated with having traffic cross and merge. The reduction in network clearing time afforded by removing merging is likely to be a function of the efficiency within which merging can be conducted. If this process is very time consuming (e.g. four-way stop), then removing intersection conflicts in a routing plan should reduce network clearing time more than in cases where intersection conflicts are managed very efficiently (e.g. adaptive signal control). In a subsequent section, we use manual capacity analysis and microscopic traffic simulation to demonstrate these various cases.

3. Model formulation

The routing problem described in this section is based on the minimum-cost flow problem (Ford and Fulkerson, 1962). The min-cost flow problem holds a central position among network optimization models because it encompasses a broad class of applications (Hillier and Lieberman, 1990). The objective is to minimize the cost of transporting all supply (source nodes) to meet all demand (destination nodes) in a capacitated network. It is a linear programming problem that can be solved optimally in a very efficient manner using the network simplex method, a streamlined version of the simplex method (Ahuja et al., 1993; Eiselt and Sandblom, 2000).

The evacuation routing problem (ERP) is an integer extension of the min-cost flow problem. The primary objective is to route vehicles to their closest evacuation zone exit. A secondary objective is to minimize the number of intersection merging-conflicts. Furthermore, the model prevents intersection crossing-conflicts. It can be specified as follows:

i	index of network nodes
$i \to j$	directed arc from node i to node j
b_i	net flow at node i
d_{ij}	distance along $i \to j$
u_{ij}	capacity for $i \to j$
M	upper bound on the number of merges

Decision variables:

x_{ij}	vehicle flow on $i \to j$
y_{ij}	$\begin{cases} 1 & \text{if the flow on arc } ij \text{ is positive} \\ 0 & \text{otherwise} \end{cases}$
z_i	the number of traffic streams that merge at node i

Evacuation routing problem:

$$\text{Minimize:} \quad Z = \sum_i \sum_j d_{ij} x_{ij} \tag{1}$$

$$\text{Subject to:} \quad \sum_j x_{ij} - \sum_j x_{ji} = b_i \quad \text{for all } i \tag{2}$$

$$x_{ij} \leqslant u_{ij} y_{ij} \quad \text{for all } i \to j \text{ that cross } k \to l \tag{3}$$

$$x_{kl} \leqslant u_{kl}(1 - y_{ij}) \quad \text{for all } k \to l \text{ crossed by } i \to j \tag{4}$$

$$\sum_j y_{ji} \leqslant z_i + 1 \quad \text{for all } i \text{ with a potential merge} \tag{5}$$

$$\sum_i z_i \leqslant M \tag{6}$$

$$0 \leqslant x_{ij} \leqslant u_{ij} \quad \text{for all } i \to j \tag{7}$$

The objective (1) is to minimize total travel distance. Constraint (2) is the standard flow conservation constraint in a network flow problem: the flow out of a node minus the flow into the node must be equal to the net flow at the node. Typically, b_i is a constant that is positive for source nodes, zero for intermediate nodes, and negative for destination nodes. In this model, b_i is positive for source nodes, zero for intermediate nodes but a variable for evacuation zone exit nodes. This allows the net outflow at exits to be resolved endogenously along with the routes. Constraints (3) and (4) prevent intersection crossing-conflicts. Constraint (5) records a merge at node i for each traffic stream above 1 that terminates at the node. The y_i variables are binary, so the z_i's can be relaxed linear variables, and they are guaranteed to have integer values. To prevent three-way merging into a single lane, an optional upper bound of 1 can be placed on the z_i variables.

Constraint (6) places an adjustable upper bound on the total number of merges allowed in a routing plan. This is the secondary objective of minimizing the number of merging conflicts represented using the constraint method for multi-objective programming (Cohon, 1978). Constraint (7) places bounds on all flow variables.

4. Lane-based network data modeling

Lane-based network data modeling is a finer level of geographic detail than typically relied upon in network flow modeling. In recent years, ITSs and other transportation applications have highlighted the need for lane-based network data models (Miller and Shaw, 2001). In the context of evacuation routing, lane connectivity is very valuable, as abstracting an intersection into a single node conceals important traffic flow details that might cause delays (Ziliaskopoulous and Mahmassani, 1996). This can lead to routing ambiguity because an intersection might be in any number of states at the lane level. Fig. 3 depicts a standard node-per-intersection network flow data model that can be interpreted in different ways. This data model cannot distinguish the upper case from the lower case. This is important because the potential traffic delay in the upper case is greater under elevated travel demand, as traffic is merging into a single lane.

To represent lane connectivity, a node-per-lane model can be used. Fig. 4 depicts a standard four-leg, single-lane approach intersection with one lane in each direction (Garber and Hoel, 1997). There are eight nodes that represent intersection entry and exit points and 12 possible turns. Thus, 12 linear arc variables (x_{ij}) are required per intersection using this data model. There are 16 crossing conflict-points, and four merging conflict points. A binary crossing matrix in Fig. 4 depicts the arcs that cross. Because arc a crossing b is the same as b crossing a, only the 1's above the diagonal are shown. Six matrix rows contain a 1, so six binary variables (y_{ij}) and six constraints would be required to represent constraint set (3). There are sixteen 1's in the matrix, leading to 16 constraints per intersection for constraint set (4). Capturing merges requires binary variables for each arc that might merge with other arcs. This leads to 12 binary variables (y_{ij}), four linear merge variables (z_i), and four constraints for constraint (5). Although six binary variables can prevent traffic stream crossings, all 12 arcs require a binary variable to detect merges.

Network data modeling can significantly affect model solution time. A representation that reduces the number of variables and constraints in a problem instance is preferable. Fig. 5 depicts more compact representations for a four-leg single-lane approach and double-lane approach intersection. This model eliminates right-hand turn arcs, which reduces the number of linear flow

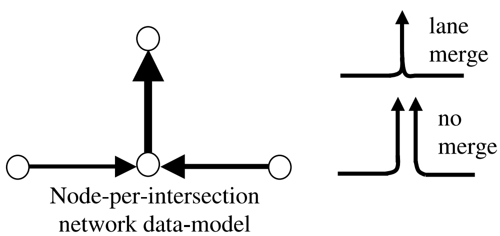
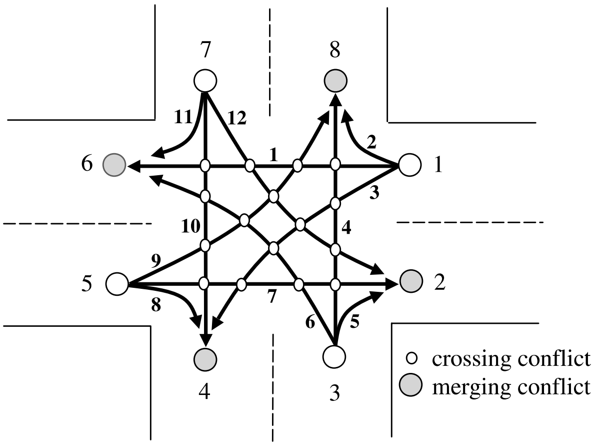


Fig. 3. A node-per-intersection network flow data model cannot represent the difference between a lane merge and non-merge.



arc	1	2	3	4	5	6	7	8	9	10	11	12
1				1					1	1		1
2												
3				1		1	1					1
4							1					1
5												
6							1		1	1		
7										1		
8												
9										1		1
10												
11												
12												

Fig. 4. A four-leg, single-lane approach data model and crossing matrix.

variables per intersection by 4. The number of binary variables remains 12 because variables must be added to the four approaching arcs in each intersection to detect merges with a right-hand turn. The number of crossing conflicts remains 16, and the number of merging variables (z_i) and constraints both remain 4. There are three potential crossing conflicts that might occur at each node. For example, at node 1 in Fig. 5, arcs 1 and 2 cross, 1 and 3 cross, and 2 and 4 cross. Overall, the reduction in variables and constraints afforded by this representation warrant its use. There are many other intersection and link types in real road networks (TRB, 1992), but they can be represented with variations of the data models shown.

An additional issue that arises in lane-based data modeling is exclusive turning lanes. Although these lanes are beneficial in normal traffic operations, they do not add any turning latitude in an evacuation routing context. In other words, given that intersections are to be placed in state of uninterrupted flow, exclusive left and right-hand turning lanes do not add any turns that would not otherwise be possible without them. For this reason, it is not necessary to add additional logical nodes and arcs to represent these lanes. This substantially reduces data-model complexity and the number of variables necessary to represent the model in equations (1)–(7).

Single-lane data models may suffice in neighborhoods and rural areas, but in many urban ERPs, a multi-lane model is required. There are a number of issues that arise in a multi-lane evacuation routing context. Central to these is how additional lanes along a link should be uti-

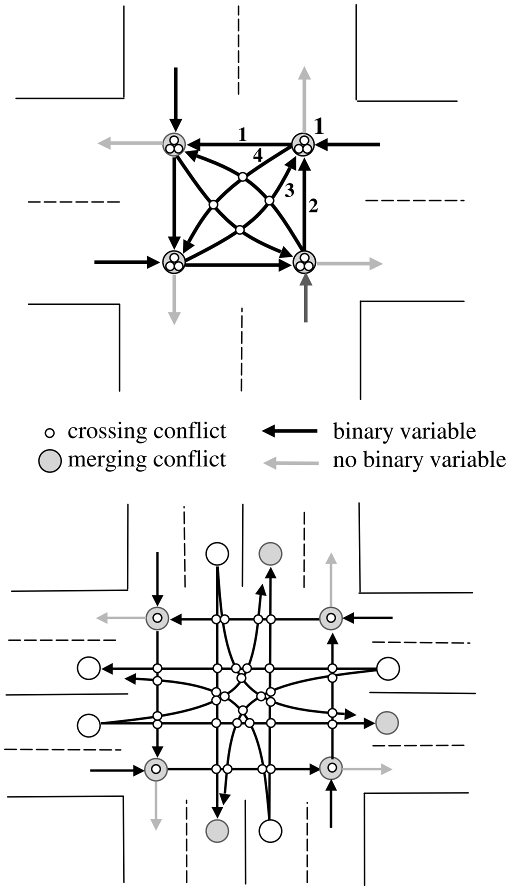


Fig. 5. Compact data models for four-leg single-lane approach and double-lane approach intersections.

lized. For example, there are three ways to enter a left (or center) lane along a link in a routing scheme. In the first case, the vehicle might enter from a driveway or other intermediate access point along a link. This can be modeled as a direct source (origin) flow into the left lane. The second entry method is to change lanes from the right lane into the left lane. If lane changing is allowed along a link, it can be represented with two additional arcs that connect the lanes. The third entry method is to make a left-hand turn at an intersection into a left lane from an intersecting leg. A standard left-hand turn arc can handle this option. Fig. 6 depicts these three cases along with the addition case of a reversible lane. Note that explicit lane-changing arcs introduce a potential crossing and two potential merging points into a model (Fig. 6b). Given that the model in (1)–(7) minimizes merging, lane changing in a routing plan will naturally be avoided if this option is used. In the reversible lane case, Fig. 6d, additional binary variables must be added to the model because only one direction is possible in a routing scheme.

Any turn that a decision maker deems possible in a real evacuation should be represented in a lane-based network data model. This is a larger set than allowed under normal network operations. For example, in an emergency, a right hand lane in a four-leg double-lane approach

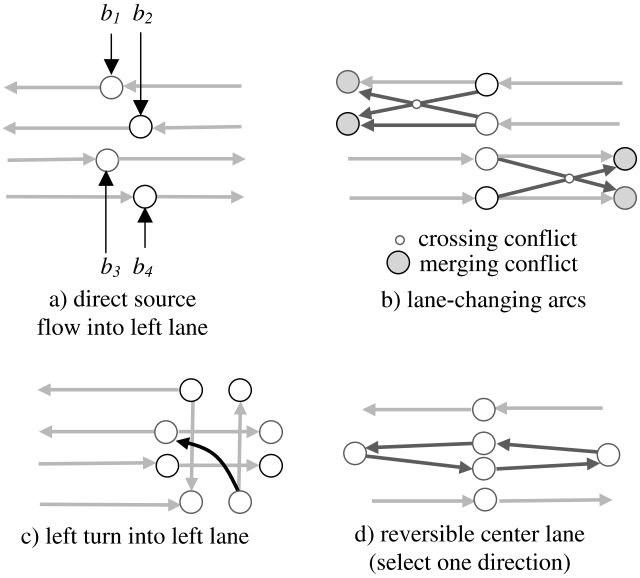


Fig. 6. Data modeling methods for utilizing higher-order lanes in a network.

intersection might be routed using a left hand turn into the right lane of the departing leg. The additional variables required to represent this class of turn make a problem instance more difficult to solve optimally. For this reason, the recommended approach is to add additional turns as needed. If an acceptable routing plan can be identified with standard traffic rules, this is the preferred option. Additional intersection turns and lane changing can be added to the data model in stages until a suitable routing plan emerges.

5. Computational experiments

5.1. Solving the model

This section demonstrates the model's output and solution time with controlled computational experiments on simple data sets. CPLEX 7.0 was used to derive optimal solutions to the model (ILOG, 2000). Fig. 7 depicts a nine-intersection evacuation zone of two-way streets represented using the compact four-approach intersection data model. The network has 60 nodes, 120 arcs, 144 potential crossing conflicts, and 24 nodes at which merging might occur. Eliminating boundary arcs and nodes entering the sample data set can reduce the number of variables and constraints in a problem instance. These arcs will never have positive flow if emergency personnel block all entrances to the evacuation zone. The 1's in intermediate nodes along links represent source flow ($b_i = 1$ in Eq. (2)). This can be interpreted as the, "presence of source flow in lane *i*." The four regions bounded by intersections might be city blocks, urban areas, or rural areas of any shape.

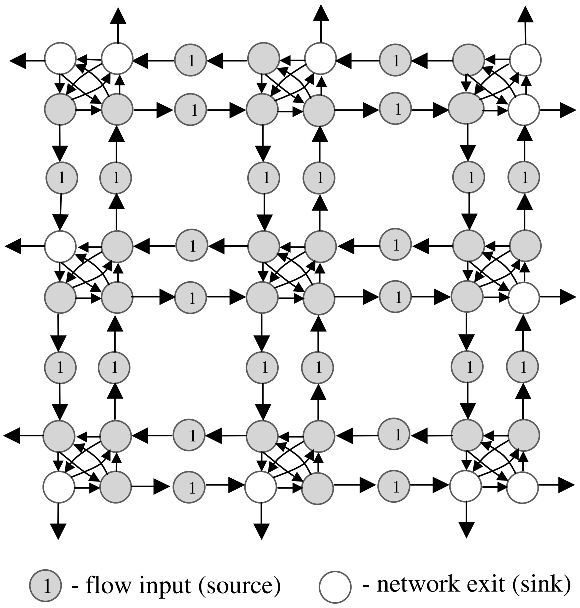


Fig. 7. A sample network with nine intersections and 12 exits.

All arcs in the sample network have unit travel distance ($d_{ij} = 1$ in Eq. (1)) and unlimited capacity ($u_{ij} = 100$ in Eq. (7)). Thus, traveling between intersections has cost 2, left turns and traveling straight through an intersection have cost 1, and right-hand turns have no cost. There are 12 exits from the network that may be selectively blocked by a hazard or emergency personnel. For example, closing the four southwest exits results in directional egress to the northeast, and closing all but the five northern most exits results in directional egress to the north.

Fig. 8 depicts routing solutions for varying degrees of merging and exit availability. There are no intersection crossing-conflicts in any plan. The routing plan in the upper left-hand corner corresponds to the shortest-distance plan for the 12-exit case (eight merges). Given eight merges, the minimum travel distance to route vehicles out of the evacuation zone is 48. Because of the regularity in the sample data set, multiple optima exist for all cases in the scenario matrix. For the upper right-hand case (no merges), 16 optima were identified. Additional criteria can be used to distinguish these solutions from one another. For example, minimizing the number of left hand turns is beneficial in a routing plan because right hand turns are easier for emergency personnel to direct. An additional constraint was added to control the maximum number of left-hand turns allowed in a routing plan using an upper bound L. The following constraint was added to the problem in (1)–(7):

$$\sum_{i} \sum_{j} y_{ij} \leqslant L \quad \text{if arc } i \to j \text{ is a left turn} \tag{8}$$

Of the 16 optima identified for the upper right-hand case using the initial criteria, only one had 4 left hand turns, the minimum. All other routing plans shown in Fig. 8 represent the solution with

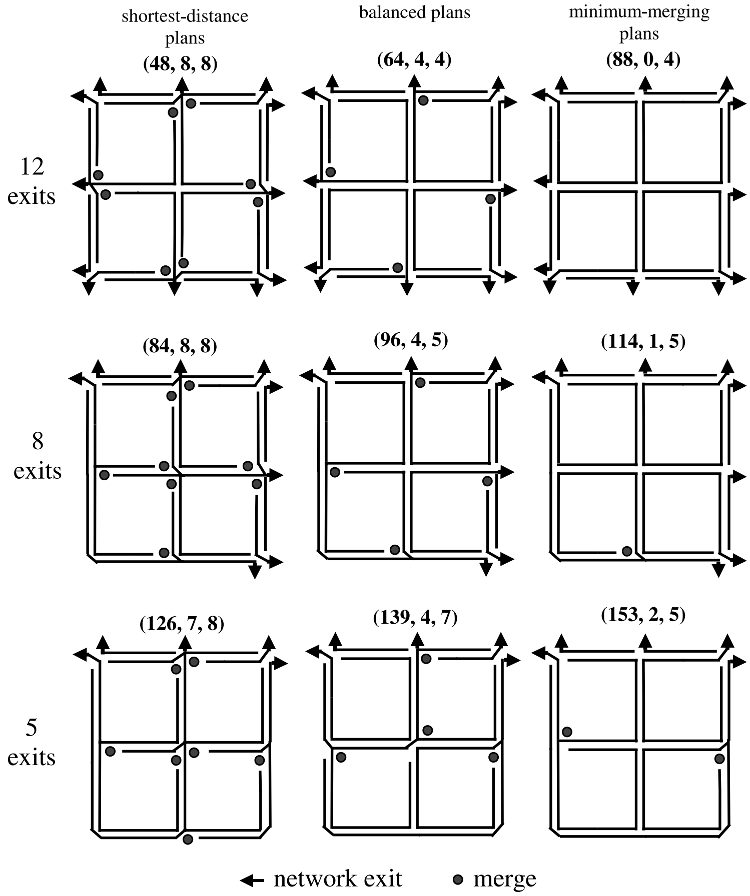


Fig. 8. Lane-based routing plans for various combinations of distance, merging, and left turns (D, M, L) for three levels of exit availability.

the least number of left-hand turns selected from the set of optima using only the distance and merging criteria. For example, the solution in the center is the eight-exit case, where up to four merges and five left-hand turns were allowed. The minimum travel distance to route all evacuees out of the area was 96 ($D = 96$, $M = 4$, $L = 5$). Reducing the number of left hand turns in this problem instance to 4 would result in a greater total travel-distance.

The trade-off between merging and travel distance in Fig. 8 can be observed across each row. For example, in the case where the hazard blocks all but the northern most exits (bottom row), if seven merges are allowed and eight left hand turns, the travel distance is 126. As the number of allowable merges is decreased to 4 and then 2, the travel distance increases to 139 and 153 because vehicles have to travel further from their closest exit to avoid merging. The plans in the right hand column represent the routing plans with minimal merging. In the 8-exit case (middle row), there was no solution with less than one merge, and in the five-exit case there was no solution with less than two merges. The plans in the left hand column represent the SEP. In this column, each vehicle travels directly to the nearest exit (given no crossing conflicts), but merging is at its

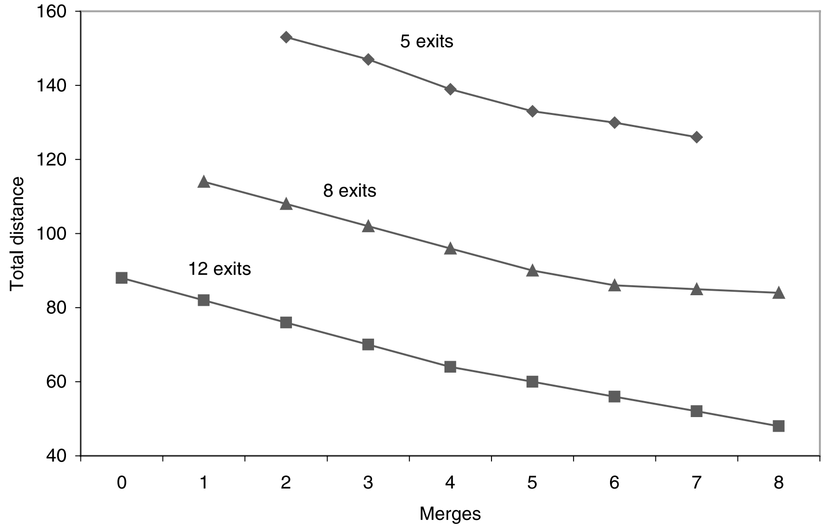


Fig. 9. The trade-off between distance and merging for three levels of exit availability.

greatest. The plans in the middle column represent a compromise between merging and travel distance. These trade-offs are quantified in Fig. 9 for the given exit scenarios.

Table 1 presents solution times for a 16-intersection (4 $\times$ 4) and 25-intersection (5 $\times$ 5) network using the same data model given in Fig. 7. The solution times are depicted as a function of the maximum number of merges and the number of available exits. In each case, the available exits were a contiguous group on the fringe of the sample network.

Solution times tend to decrease as the number of exits is decreased and egress is reduced, but this is not universal. This can be seen in the column means at the bottom of Table 1a and b. Furthermore, solution times tend to increase and then decrease as the number of allowable merges is increased from the minimum. This can be seen in the row means. In general, the computational burden of identifying solutions with few merges or many merges is lower than that of identifying solutions with a moderate amount of merges, but this is also not universal.

5.2. Assessing network clearing time

5.2.1. Manual capacity analysis

Manual capacity analysis was used to compare the relative efficiency of the various evacuation routing plans in the nine-intersection network. The performance measure that we selected to represent efficiency is network clearing time. This measure places greater importance on transportation network performance than human behavior, but this is our focus. To simplify the analysis, we assumed that network clearing time could be estimated using one side of the sample network because the major routing feature in each plan is repeated on each side. In other words, all four sides should take approximately the same time to clear.

On a given side of each routing plan, there is a principle intersection that determines how long it will take to clear the network. Each of these intersections has a critical volume-to-capacity ratio

Table 1
The 16 intersection (a) and 25 intersection (b) solution time tests (in seconds)

Merges	Number of exits						Mean time
	16		10		4		
	Time	Iterations	Time	Iterations	Time	Iterations	
(a) 16-intersection sample network (4 × 4)							
1	3	1215	Infeas.	Infeas.	Infeas.	Infeas.	3
6	10	12,459	33	40,302	8	6359	17
11	59	85,866	13	14,301	2	1464	25
16	5	5762	6	5360	3	2679	5
21	1	373	2	995	1	522	1
Mean	16	21,135	14	15,240	4	2756	
(b) 25 intersection sample network (5 × 5)[a]							
	20		12		4		
4	199	191,544	Infeas.	Infeas.	Infeas.	Infeas.	199
10	400	395,974	149	131,643	Infeas.	Infeas.	275
16	94	88,106	33	22,899	67	62,638	65
22	14	9904	10	4799	16	11,622	13
28	161	152,988	2	1063	9	5379	57
34	4	809	4	1590	2	1135	3
40	1	306	1	596	1	889	1
Mean	125	119,947	33	27,098	19	16,333	

Platform: Sun Enterprise 3500 (366 MHz) with 1024MB of RAM running Solaris 2.6.
[a] MIP termination gap of 2% for 25 intersection problems.

$(v/c)_c$ that represents the approach with the greatest demand to capacity. If the intersection is signalized, then the capacity of each approach is given by (TRB, 1992):

$$c_i = s_i(g_i/C) \tag{9}$$

where s_i, is the saturation flow capacity of approach i, g_i is the effective green time for approach i, and C is the cycle length. In practice, many factors enter in to the calculation of the saturation flow capacity, but our analysis is theoretical, so we will forgo discussing these factors to simplify the comparison.

Fig. 10 depicts the major routing feature of the three plans in black. When the critical v/c ratio of the principle intersection exceeds 1, it can be used to estimate the time that it will take to clear the area for a given plan. If the critical v/c ratio is less than or equal to 1, there is no significant delay, and network clearing time is slightly more than the time horizon t within which vehicles enter the network. When the ratio exceeds 1, it can be multiplied by the time horizon t to estimate network clearing time T:

$$T = (v/c)_c \cdot t \tag{10}$$

For example, assume that each single-lane approach in the study area has a saturation flow rate s_i of 1800 vph regardless of the movement. In the case of the minimum-merging plan, there are no

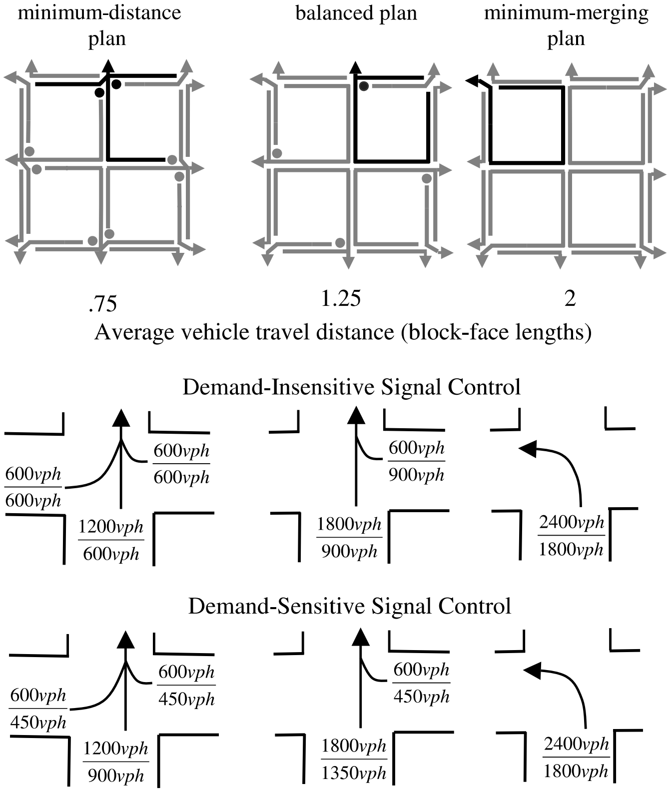


Fig. 10. The major routing feature in each of three evacuation routing plans (in black) and example intersection capacities for the principle intersection under demand-insensitive and demand-sensitive signal timing.

merges, so vehicles entering from four block faces have a capacity of 1800 vph in the critical approach because there is no need to interrupt the flow (i.e. protected left-hand turn out of the area). If vehicles enter the network at a rate of 600 vph from each of the four block faces for 15 min, then the estimated network clearing time for the routing plan would be (row 1 in Table 2):

$$(600\ \text{vph} \cdot 4)/1800\ \text{vph}) \cdot 15\ \text{min} = 1.33 \cdot 15 = 20\ \text{min} \tag{11}$$

In the minimum distance and balanced plans, there are varying degrees of merging. For this reason, the capacity of the intersections depends on the type of intersection control. For example,

Table 2
Network clearing time calculations using manual capacity analysis

Routing plan	Vph per origin	Origin zones	Signal timing	Capacity (vph)	v/c	Time horizon	Clearing time (min)
Minimum merging	600	4	None	1800	1.33	15	20.0
Balanced	600	3	Equal	900	2.00	15	30.0
Minimum distance	600	2	Equal	600	2.00	15	30.0
Balanced	600	3	Proportional	1350	1.33	15	20.0
Minimum distance	600	2	Proportional	900	1.33	15	20.0

if green time is divided equally between two protected turns, each approach will have a capacity of approximately 900 vph because they depart into a lane with a capacity of 1800 vph (Fig. 10). In the balanced plan, the critical v/c ratio occurs on the approach that has vehicle flow arriving from three block faces. The network clearing time would then be 30 min, as shown in row 2 of Table 2. For the shortest distance plan, where two block faces rely on the critical approach and three turns must be merged (600 vph each), the network clearing time is also 30 min, as depicted in row 3 of Table 2. In this simple comparison with relatively inefficient intersection control, the minimum-merging plan has the lowest network clearing time of 20 min. The shortest distance plan and balanced plan take 50% longer at 30 min.

The relative efficiency of the plans is sensitive to the capacity of the principle intersection. For example, if intersection capacity is improved so that the green time is *proportional* to the arriving demand on each approach (i.e. demand sensitive), then the capacity of the critical v/c ratio in the balanced plan would be 0.75×1800 vph $= 1350$ vph because three of four block faces use the critical approach (Fig. 10). Similarly, the capacity for the critical approach in the shortest-distant plan under proportional timing would be 0.5×1800 vph $= 900$ vph because two of four origin zones rely on the critical approach. Using the same demand as in the prior examples, the network clearing time for the balanced and shortest-distance plans under proportional signal timing is 20 min. This is depicted in rows 4 and 5 of Table 2.

Therefore, when (protected) merging is controlled in a relatively efficient manner, the three plans have the same network clearing time: 20 min. This means that the benefit of minimizing merging in a routing plan depends upon the efficiency within which merging can be performed. If intersection control is very inefficient at merging traffic streams, as in the case of four-way stops or flashing-red signals from a power failure, then a routing scheme that minimizes merging will likely result in a lower total network clearing time. However, if the signal timing is highly demand-sensitive, as in a fully actuated signal, then the benefit of avoiding merging is likely to be negligible.

5.2.2. Microscopic traffic simulation

Microscopic traffic simulation (microsimulation) was also used to compare the efficiency of the sample routing plans generated in the prior section. Fig. 11 depicts the nine-intersection microsimulation study area with 24 origin zones and 12 destination zones. Our intent was to compare network clearing times for the plans that the routing model generates under various demand and intersection-capacity scenarios. Microsimulation allowed us to compare the routing plans to the *no routing* plan (null case) in addition to increasing our confidence in the simple manual capacity calculations presented in the prior section. Paramics™ by Quadstone™ was used to run the experiments (Cameron and Duncan, 1996).

Seven evacuation routing cases were compared. The first, the *no-routing* case, assumed that flows were not channeled at intersections. Drivers departing from each of the 24 origin zones randomly selected one of the 12 destination zones. This is a near worst-case destination-choice behavior on the part of evacuees that results in many crossing and merging conflicts. This case was included as a theoretical baseline for assessing the benefit of implementing any routing plan with flow channeling at intersections. The no-routing case was divided into signalized (equal timing) and uncontrolled intersections resulting in two no-routing scenarios. The three lane-based routing

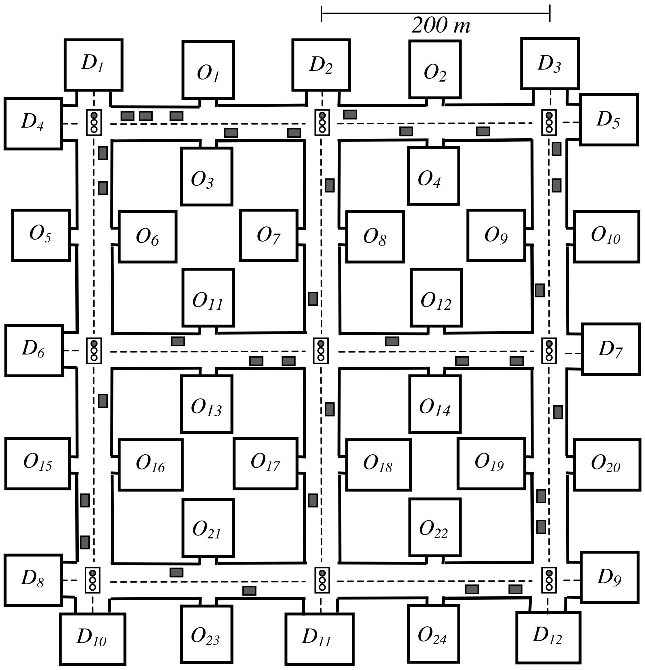


Fig. 11. The network for the microsimulation experiments.

plans evaluated were the minimum-distance, balanced, and minimum-merging plans depicted in the top row of Fig. 8.

For the minimum-distance and balanced routing plans, intersection control was introduced to safely control merging. For these cases, we developed *demand-insensitive* and *demand-sensitive* intersection signal timings. In the demand-insensitive case, the green time was equal (and fixed) for each approach regardless of the demand or routing plan. This represents a relatively inefficient (low capacity) intersection. In the demand-sensitive signal-timing case, the green time for each approach was proportional to the vehicle demand on that approach for a given plan. This is representative of a fully-actuated signal or very efficient manual traffic-direction. In both signal-timing cases, all turns were protected (i.e. no left-hand turns on green or right-hand turns on red), so green time rotated through the various approaches. No green time was allocated to an approach entering the evacuation zone. This generated four more evacuation-routing scenarios. The last case evaluated was the minimum-merging plan with no crossing or merging conflicts. The chief capacity constraint in this plan was the saturation flow capacity of a left-hand turn out of the evacuation zone.

For experimental simplicity, the departure rate for each origin zone was uniform. This is unrealistic, but we assumed that the departure rate would not affect the *relative* network clearing times of the routing cases. The time horizon for demand input into the network was 15 min. So, for the case where the volume is 120 vph per origin zone, the simulation model uniformly input 30 vehicles per origin zone into the network over 15 min. Each simulation scenario was run 10 times and the mean network clearing time was calculated. Network clearing time was defined as

Table 3
Mean network clearing time for the sample network using microsimulation

Routing plan	AVD	Cross	Merge	Signal	Vph per origin zone					% Decrease	Manual capacity	
					120	240	360	480	600	600	600	Difference
None		Yes	Yes	None	16.0	21.6						
None		Yes	Yes	Equal	16.8	23.2	35.0	47.1	54.9			
Minimum distance	0.75	No	Yes	Equal	15.8	15.9	20.1	26.6	32.9	40.1	33.3	0.4
Balanced	1.25	No	Yes	Equal	15.5	15.6	20.2	26.3	32.8	0.3	33.3	0.5
Minimum distance	0.75	No	Yes	Proportional	15.8	16.1	16.3	18.2	22.3	32.0	22.2	0.1
Minimum merging	2.00	No	No	None	15.6	15.7	15.7	18.1	22.3	0.3	22.2	0.1
Balanced	1.25	No	Yes	Proportional	15.5	15.9	16.1	17.8	21.9	1.8	22.2	0.3

the point when the last vehicle cleared the network, and vehicles departing from an origin zone were restricted from crossing street centerlines in all cases.

Table 3 depicts the result of the simulation experiments. The columns in the table are the routing plan, average vehicle distance (AVD) in block faces, existence of crossing and merging conflicts, signal timing, mean network clearing times (in minutes) as a function of travel demand per origin zone, and the percent decrease from the prior plan (600 vph case). For the no-routing case with uncontrolled intersections, gridlock occurred at input volumes greater than 240 vph per origin zone. Network clearing time is not reported for these cases. The rows are sorted in descending order by network clearing time for the 600 vph per origin zone case.

The most significant decrease in network clearing time occurred in shifting from the *no routing* case with equal signal timing to a routing plan that prevents traffic crossing conflicts (i.e. 40% decrease for the 600 vph per origin zone case). The next significant decrease occurred in shifting to either a demand-sensitive signal timing strategy or the minimal merging plan (i.e. additional 32% decrease in 600 vph case). These results confirm the manual capacity analysis results. Namely, minimizing merging-conflicts has approximately the same network clearing time as a shortest-distance or balanced plan when intersection control is relatively efficient at merging traffic. As in the manual capacity analysis, if the intersections in the evacuation zone are very efficient in conducting merges, then the benefit of removing merging conflicts becomes negligible. However, if the critical intersections in the evacuation are four-way stops, or they have been reduced to four-way stops because the signals are not working due to a power failure, then a minimal merging plan may hold the most promise for reducing network clearing times. In any case, channeling flows to remove crossing conflicts appears to be beneficial regardless of the level of merging.

To verify that the manual capacity estimates provide similar results to the microsimulation experiments, we calculated the time that a scenario in Table 3 should take for the 600 vph case using the manual capacity formula. The saturation flow capacity of a lane in Paramics™ is approximately 1620 vph. For example, in the minimum-distance plan with equal signal timing (i.e. demand insensitive), the estimate would be:

$$((600 \text{ vph} \times 2)/(1620 \text{ vph}/3)) \times 15 \text{ min}) = 2.2 \times 15 = 33.3 \text{ min} \tag{12}$$

This was within 30 s $(33.3 - 32.9 = 0.4)$ of the estimated network clearing time using microsimulation. The far right of Table 3 depicts similar manual capacity calculations for the 600 vph per origin zone case. The greatest difference (diff.) between the mean network clearing time using microsimulation and the manual capacity calculation was 30 s (0.5 min).

Although this was a very simple experiment, it supports the theory that reducing the number of merging conflicts in a routing plan reduces network clearing time when traffic volumes are high and intersections capacities are relatively low. We did not find the minimum-distance plan to be the quickest alternative under light to moderate traffic volumes, as predicted in Fig. 2. Under light traffic volumes, the difference between the plans was insignificant. In general, if traffic managers do not foresee merging at intersections as causing significant delays, then a shortest-distance plan is the most straightforward option. If intersection delays due to merging are expected to be significant, then the opportunity arises to minimize merges as a means for reducing these delays. However, more tests need to be performed to verify the cases where this holds, particularly in more complex networks.

6. Lane-based evacuation routing in Salt Lake City, Utah

From 1995 to 2000, four notable evacuations occurred in downtown Salt Lake City. Two were due to armed assailants, one was the result of a tornado, and one was a bomb threat at the State of Utah Office of the Regents that was later determined to be a diversion for a bank robbery (Hoard, 2001). All of the evacuations were conducted in the same manner. An emergency planning zone containing the incident was blocked to entering traffic by police and designated a pedestrian evacuation zone (PEZ). People in this zone were instructed to proceed out of the zone on foot, abandoning all vehicles. Traffic surrounding this internal zone was instructed to leave the area of its own accord. This represents the surrounding vehicle evacuation zone (VEZ). In all four evacuations, traffic gridlock occurred on the fringe of the PEZ, as drivers were unable to use the sub-network blocked by police to leave the area. An approach to alleviating the gridlock surrounding the PEZ is to develop a lane-based evacuation routing plan for the surrounding intersections. This would require a team of emergency personnel to direct traffic, but it holds the potential to reduce intersection delays and network clearing time. The model described in this paper can be applied to the problem of routing vehicles out of this external VEZ.

The study area is a 20-intersection area of downtown Salt Lake City where the four evacuations occurred. This area includes prominent office buildings, hotels, an events center, a conference center, tourist attractions, a shopping mall, light rail stations, restaurants, and numerous parking lots above and below ground. The area is capable of generating many thousands of vehicle trips in a daytime evacuation. It is also clear from prior evacuations that the v/c ratios for all intersection approaches are much greater than 1. This is the basis for applying the routing model presented in Section 3. Furthermore, it is common for power failures to render signals unusable in a disaster. For this reason, removing merging conflicts in a routing plan is also likely to be beneficial because there is no efficient manner in which to conduct merges under this scenario.

A first step in applying the routing model is developing a digital representation of the area of interest. Digital lane-level information is scarce. Most network information is a street-centerline representation with the number of lanes represented as an attribute of each arc in the database (Miller and Shaw, 2001). One method for acquiring lane connectivity is to use high-resolution imagery. An air-photo mosaic was commissioned for the Salt Lake City vicinity at 1-foot resolution to plan for the Winter Olympics in 2002. This information was used and later field-checked to generate a lane-based network model.

Fig. 12 is a schematic of the data model. This representation exaggerates the scale of the intersections to reveal lane-turning connectivity. The area is comprised of 20 intersections joined by 48 lane groups (i.e. adjacent lanes between two intersections with the same direction of travel) and 111 individual lanes. The node-arc representation of the area has 314 nodes, 443 directed arcs, and 607 crossing conflicts. Thirty-five nodes on the fringe of the network represent evacuation zone exits. Each lane is represented using two directed arcs with a node at the lane midpoint. The midpoint node is assigned a b_i value of 1 in constraint (2). This represents the presence of origin flow in the lane. Volume inputs are not represented because we know that the v/c ratio for all intersection approaches will be well above 1. Also, as stated, our goal is to develop a routing model that can be applied without this information because emergency managers do not have time to collect it in an emergency.

Lane capacities in the study area were assumed to be unlimited, so the node-arc model has the same format as in Fig. 7. Drivers entering the network from parking lots, alleys, and other entry points along a link are free to seek any starting lane. However, once a driver selects a lane, emergency personnel at each intersection determine the vehicle's route. Given that the lane groups connecting all intersections are the same length in the study area, all arcs were assigned unit length. Therefore, traveling between intersections has cost 2 (two arcs), left turns and traveling straight through an intersection have cost 1, and right turns have no cost.

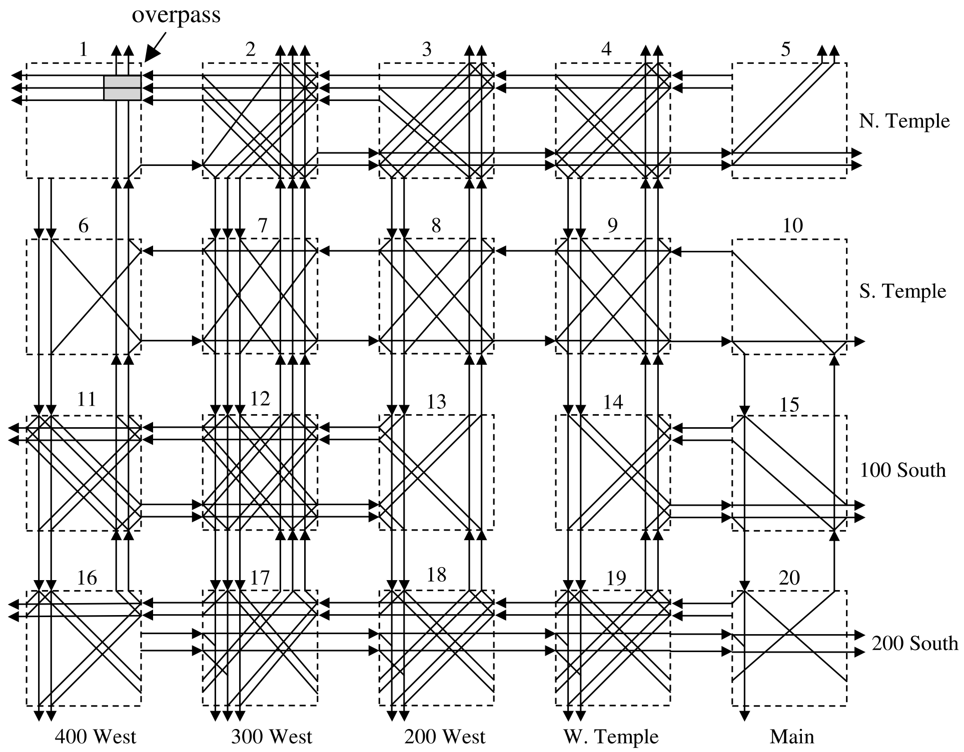


Fig. 12. A traffic-lane schematic for the 20-intersection Salt Lake City study area.

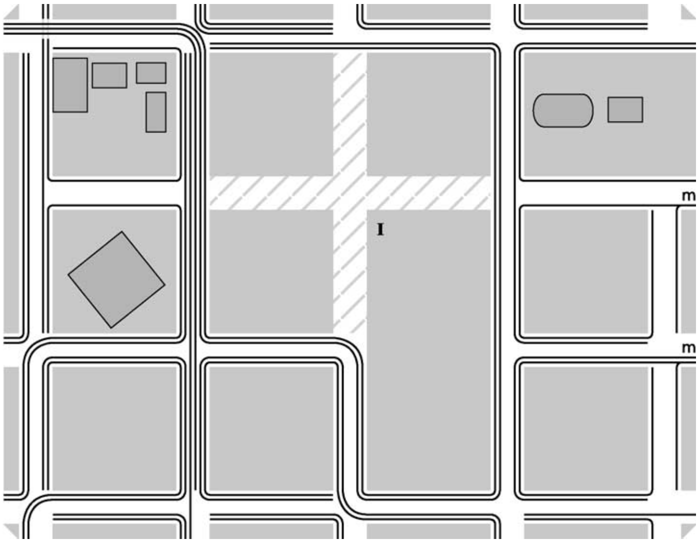


Fig. 13. A lane-based evacuation routing plan, given a pedestrian evacuation zone.

As noted, the four recent evacuations in downtown Salt Lake City involved an internal, contiguous pedestrian evacuation zone. An example PEZ was established under the assumption that an incident is in progress at the intersection of 200 West and South Temple ('I' in Fig. 13). The four links entering this intersection are considered blocked by police and unusable by vehicles. The boundary of a PEZ is the line where emergency managers deem that the incident no longer poses a threat to vehicles. A bomb threat is a good example of such a hazard. Fig. 13 depicts a routing plan given the defined 1-block PEZ. This plan limits the number of merges to 2, the minimum, and the number of left turns to 11. For this reason, some vehicles would be routed significantly further than their closest exit to avoid merging, left turns, and crossing conflicts.

As complex as the overall routing scheme appears, the focus should be the turns at each intersection. This is where route direction occurs. The actual route that a vehicle will take to leave the area emerges as the result of controlling lane-level turns at each intersection approach. Fig. 14 depicts the trade-off between merging and total distance for the study area given the PEZ. Relaxing the number of merges from two to four improves the balance between merging and travel distance. Fig. 15 depicts a routing plan where the maximum number of merges is 4 and the number of left-hand turns is 17. This plan also requires evacuees to travel further than their closest exit in some cases. If this is not acceptable, then M in constraint (6) can be increased.

This raises the question of the best position on the non-inferior trade-off curve between total travel-distance and merging. In general, if traffic congestion is expected to be severe and intersections are very inefficient at merging traffic (e.g. flashing red or four-way stop), then a plan with minimal merging is likely to perform best. In this type of plan, some vehicles will be routed away from their closest exit, but intersection delays will be minimized with no crossing conflicts, minimal merging conflicts, and minimal left turns. If traffic volumes are expected to be moderate, and the situation is less urgent, then traffic managers can opt for a plan with moderate merging. If traffic congestion is not a factor, then a shortest-distance routing plan is the most straightforward

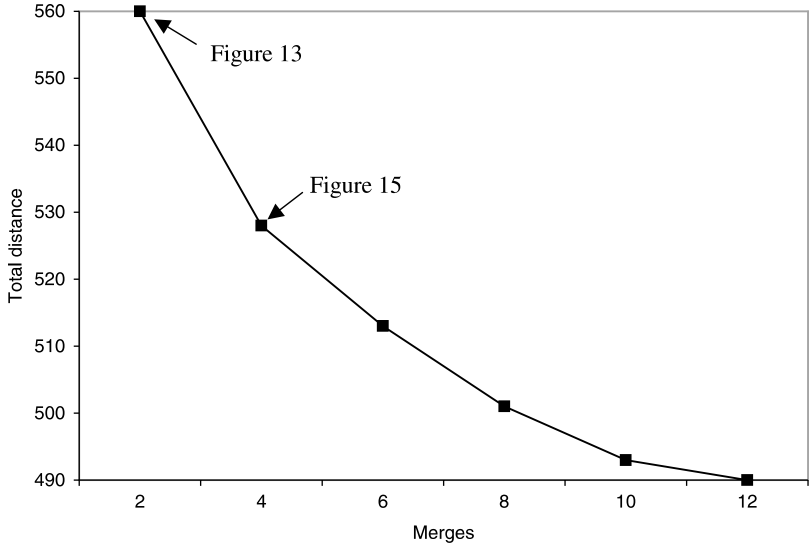


Fig. 14. The trade-off between merging and distance, given a pedestrian evacuation zone.

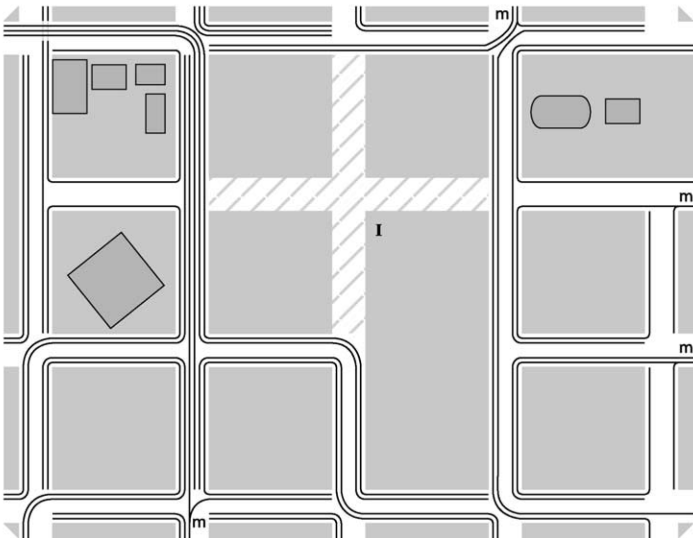


Fig. 15. A routing plan with an improved balance between merging and total travel-distance.

approach. These are only guidelines, and experienced emergency managers must decide which routing plan is best in a unique hazardous situation.

7. Discussion

Many issues arise in implementing a lane-based routing plan. In general, a traffic manager or traffic operations center would run the model at the time of a recognized threat or event. The routing plan would then be relayed (e.g. Palm Pilot) to emergency responders in the field who would set up barriers (e.g. pylons, vehicles, or temporary barricades) to prevent traffic from entering the evacuation zone and channel flows at critical intersections. This would require a significant amount of coordination among emergency personnel as well as cooperation from evacuees. If there are not enough personnel to direct traffic at every intersection at once, then a plan can be implemented in stages. For example, stage 1 might be preventing traffic from entering the evacuation zone and directing traffic on the fringe to turn right and exit. This pattern is evident in many plans generated by the model (Figs. 13 and 15); traffic on the fringe is directed to turn right out of the zone at the first opportunity. This alone could improve the situation, as vehicles could not turn toward the center of the zone. As each intersection is placed under control and personnel become available, they can move toward the center of the evacuation zone, setting up flow-channeling patterns at the most important intersections until most (or all) intersections are functioning as uninterrupted flow facilities. In many contexts, it may not be necessary to channel flows at all intersections. The most critical points are major intersections where v/c ratios are expected to be significantly above 1.

The application in the prior section did not require accurate vehicle input and lane capacity estimates to generate efficient evacuation routing schemes. A binary parameter was used to define the presence of vehicle flow in each lane, and lane capacities were considered unbounded. This is possible because the definition of "efficient" in the routing model is couched more in terms of potential traffic stream interactions at intersections than in accurately minimizing the distance that all vehicles will travel. For this reason, the model is less sensitive to these input values than the max-flow and min-cost models (Dunn and Newton, 1992; Yamada, 1996). In special cases, like the upper right-hand plan in Fig. 8 ($M = 0, L = 4$), there is only one solution regardless of the vehicle source flow volumes. In other words, any values can be entered for source flow b_i constants in (2), and the optimal routing scheme will be the same. This is only true when merging and left hand turns are constrained to a minimum and link capacities are not restricted. When merging and left hand turns are allowed to increase in the name of minimizing total evacuee travel distance, the vehicle input values will affect the locations where merging and left-hand turns occur in a routing scheme. This is the case because the model minimizes the total distance that all evacuees will travel, and the amount and locations of merging affect this distance. Therefore, the detail and accuracy of the population input estimates increases in relevance as merging and left-hand turns are allowed to increase.

Our focus in this paper is short-term, on-the-fly evacuation route planning when vehicle inputs are not available. In longer-range planning, a simple improvement in these estimates would be to examine the block faces (or other geographic input area) in the study area to determine which have large parking lots and which might result in no entering traffic. The b_i source flow values could be adjusted to model vehicle flow in a relative fashion like, "Block face *a* will likely result in three times the vehicle input of block face *b*." If block face *a* and *b* both have three lanes, each lane in a could be assigned a source flow value of 3, and each lane in block face *b* could be assigned a source flow value of 1. Further refinement of these values could be achieved with a detailed parking study. Similarly, lane capacity estimates could be improved if lanes in the area of interest vary substantially in this regard. The main point is that accurate vehicle trip and lane capacity inputs for the proposed routing model, though valuable, are not required to generate potentially useful evacuation routing schemes. This is beneficial because these data are problematic to collect in an emergency.

8. Conclusion

This paper presented a network flow model for identifying optimal lane-based evacuation routing plans in a complex road network. The potential for lane-based route modeling to reduce traffic delays at intersections in evacuations is promising. There are many hazardous areas with relatively poor access that do not have an evacuation plan (Church and Cova, 2000; Cova and Johnson, 2002). These communities would likely benefit from an application of this routing model. The central assumption underlying the model is that placing intersections in a state of uninterrupted flow and minimizing merging will result in fewer traffic delays and lower network evacuation clearing times under moderate to heavy volumes. This can require the cooperation of evacuees and many emergency personnel if there are a large number of intersections to control. For this reason, the model presented in this paper may identify routing plans with complexity beyond that which can be implemented either from a management perspective or that of evacuee compliance.

The microsimulation results indicate that channeling flows at intersections to remove crossing-conflicts can significantly decrease network clearing time over no routing plan (i.e. random destination choice). A 40% reduction was reported, but this amount is likely to vary depending on the road-network context and scenario. A shortest-distant routing plan has no crossing-conflicts but may involve a significant amount of merging. The benefit of channeling flows at intersections to remove merging depends on traffic volumes and the efficiency within which merging can be performed. If this process is very inefficient and intersection v/c ratios exceed 1 (e.g. four-way stop or flashing red signals from a power failure), then a plan with minimal merging can further decrease network clearing time. An additional 32% reduction was reported, but again, this amount is likely to vary depending on the road-network context and scenario. If merging can be conducted in a very efficient manner, as in the case of demand-sensitive signal control, then reducing the amount of merging in a routing plan appears to have little or no benefit. In this case, a shortest-distance plan would be the most straightforward option.

There are a number of directions to pursue in the context of this research. Reversing the direction of lanes was briefly discussed but not tested, and their role in evacuation routing is potentially valuable. Heuristic algorithms to solve larger lane-based routing problems in less time are also needed. There is no limit to the time pressure that emergency managers might be under to route traffic out of an area in an emergency. The model in this paper might also serve as the centerpiece for an evacuation routing decision support system. This would involve extensive collaboration with emergency management experts to develop a system to meet their needs. Finally, although the Cerro Grande Fire evacuation provided evidence for the effectiveness of lane-based evacuation routing, more studies on this type of routing are needed.

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